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Quantum Transition of Two-Level System in a Parabolic Quantum Dot

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Abstract The time evolution of the quantum mechanical state of an electron is calculated by using variational method of Pekar type on the condition of electric-LO phonon strong coupling in a parabolic quantum dot. We obtained the eigen energies of the ground state and the first-excited state, the eigen functions of the ground state and the first-excited state this system in a quantum dot may be employed as a two-level quantum system-qubit. The supposition electron is in system's ground state in the initial time, the electron transit from the ground state to the excited state in presence of an electric field F along the x axis. The results indicate that the electron transition probability and the oscillation period increase with decreasing the electron-LO-phonon coupling constant, increasing the electric field and the confinement length.

Keywords Quantum dot · Quantum information · Quantum transition

1 Introduction

Recent year, the quantum computation and the quantum information's research have aroused the widespread interest among physicists from areas ranging from atomic physics, optics, to various branches of condensed matter physics. The key thrust behind the rush toward a working quantum computer (QC) is the development of a quantum algorithm that can factorize large numbers exponentially faster than any available classical algorithm. This exponential speedup is due to the intrinsic quantum parallelism in the superposition principle and the

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unitary evolution of quantum mechanics. It implies that a computer made up of entirely by quantum mechanical parts (qubit), whose evolution is governed by quantum mechanics. The two-level system is usually employed as the elementary unit for storing information [1–4]. Quantum computation will be realized by the laws of quantum mechanics. Several schemes have been proposed for realizing quantum computer in recent years [5–9]. In order to show the advantage of quantum computer over the most classical computer, quantum computer need to be composed of at least thousands of qubits to be feasible. Consequently, it is clear that quantum computer with a large significant number of qubits would be more realizable in solid-state systems. Therefore, it is one of the most popular solid-state quantum information research fields that qubits can be realized by solid-state devices. Many schemes have been proposed for researching quantum dot and have many kinds of contents, but they are in the initial research stage at present.

In this article, we obtained the eigen energies and their relevant eigen wavefunctions of the ground and the first-excited state of an electron in a parabolic quantum dot (QD) using the Pekar variational method considering the parabolic potential in the electron-LO-phonon strong-coupling region. A single qubit can be envisaged as this kind of two-level quantum system in a QD. For this single-electron QD qubit, Li et al. [5] presented a kind of parameter-phase diagram schemes and defined the parameters region for the use of the an InAs/GaAs as a two-level quantum system. We have obtained the electron transition probability oscillates with a period between the ground and the first-excited state in presence of an electric field F along the x axis. The relations of the period of oscillation and the electron transition probability on the electric field, the electron-LO-phonon coupling constant and the confinement length are derived. Our results should be meaningful for designing the solid-state implementation of quantum computing both theoretically and experimentally.

2 Theoretical Model

We consider the system which the electrons are bounded by the parabolic potential. The electrons are much more confined in one direction (taken as the Z direction) than in other two directions. Therefore, we shall confine ourselves to considering only taking into account the effect of electron and LO-phonon only considering electron moving on the X–Y plane. We assume that the confining potential in a single QD is parabolic,

$$V_{(\rho)} = \frac{1}{2} m^* \omega_0^2 \rho^2, \tag{1}$$

where the band mass of electron is m^*, ρ is the coordinate vector of a two-dimensional and ω_0 is the confinement strength. The Hamiltonian of electron-phonon system in presence of an electric field F along the x axis can be written as followed:

$$H = H_0 + H', \tag{2}$$

$$H_0 = -\frac{\hbar^2}{2m^*}\nabla_\rho^2 + \frac{1}{2} m^* \omega_0^2 \rho^2 + \sum_q \hbar\omega_{LO} b_q^+ b_q + \sum_q (V_q e^{iq\cdot r} b_q + h\cdot c), \tag{3}$$

$$H' = -e^* F x, \tag{4}$$

where H_0 is the imperturbation of the Hamiltonian of electron-phonon system, H' is the perturbation which is very small compared with H_0, $b_q^+(b_q)$ is the creation (annihilation)

operator of bulk LO-phonon with the wave vector $q(q = q_{\parallel}, q_{\perp})$, $\mathbf{r} = (\rho, z)$ is the coordinate of the electron, e^* is the electron charge.

$$V_q = i(\hbar\omega_{LO}/q)(\hbar/2m^*\omega_{LO})^{1/4}(4\pi\alpha/V)^{1/2}, \tag{5}$$

$$\alpha = (e^2/2\hbar\omega_{LO})(2m^*\omega_{LO}/\hbar)^{1/2}(1/\varepsilon_\infty - 1/\varepsilon_0). \tag{6}$$

Using the Lee-Low-Pines transformation to (2)

$$U = \exp\left[\sum_q (f_q b_q^+ - f_q^* b_q)\right], \tag{7}$$

where f_q will be treated as a variational function, we have

$$H_0' = U^{-1} H_0 U. \tag{8}$$

Supposing that the Gaussian function approximation is valid in the ground-state of electron-phonon system by variational method of Pekar type to be [10]

$$|\varphi_0\rangle = \frac{\lambda}{\sqrt{\pi}} \exp\left(-\frac{\lambda^2\rho^2}{2}\right)|\xi(z)\rangle|0_{ph}\rangle, \tag{9}$$

where λ is the variational parameter, since the electrons are much more strongly confined in Z direction than in other two directions and considered to be confined in a infinitesimally narrow layer, so $\langle\xi(z)|\xi(z)\rangle = \delta(z)$. And $|0_{ph}\rangle$ is unperturbed zero phonon state which satisfies $b_q|0_q\rangle = 0$. We then obtain the electron ground-state energy is the following form

$$E_0(\lambda) = \langle\varphi_0|H_0'|\varphi_0\rangle = \frac{\hbar^2\lambda^2}{2m^*} + \frac{\hbar^2}{2m^*\lambda^2 l_0^4} - \frac{1}{2}(2\pi)^{1/2}\alpha\hbar\omega_{LO}r_0\lambda. \tag{10}$$

The ground state energy of electron in a parabolic QD can be written as

$$E_0(\lambda_0) = \frac{\hbar^2\lambda_0^2}{2m^*} + \frac{\hbar^2}{2m^*\lambda_0^2 l_0^4} - \frac{1}{2}(2\pi)^{1/2}\alpha\hbar\omega_{LO}r_0\lambda_0, \tag{11}$$

where $l_0 = (\hbar/m^*\omega_0)^{1/2}$, $r_0 = (\hbar/2m^*\omega_{LO})^{1/2}$, l_0, r_0 is the confinement length and the polaron radius, respectively.

Similarly, the trial wave-function of electron-phonon system in the first-excited state may be chosen as [10]

$$|\varphi_1\rangle = \frac{\lambda^2}{\sqrt{\pi}}\rho \exp\left(-\frac{\lambda^2\rho^2}{2}\right)\exp(\pm i\phi)|\xi(z)\rangle|0_{ph}\rangle. \tag{12}$$

This satisfies the following relations:

$$\begin{aligned} \langle\varphi_0|\varphi_1\rangle &= 0, \\ \langle\varphi_1|\varphi_1\rangle &= 1. \end{aligned} \tag{13}$$

We can obtain the energy in the first-excited state by using the $E_1(\lambda) = \langle\varphi_1|H_0'|\varphi_1\rangle$

$$E_1(\lambda) = \frac{\hbar^2\lambda^2}{m^*} + \frac{\hbar^2}{m^*\lambda^2 l_0^4} - \frac{11}{32}(2\pi)^{1/2}\alpha\hbar\omega_{LO}r_0\lambda. \tag{14}$$

The first-excited state energy of electron in a parabolic QD can be written as

$$E_1(\lambda_0) = \frac{\hbar^2\lambda_0^2}{m^*} + \frac{\hbar^2}{m^*\lambda_0^2 l_0^4} - \frac{11}{32}(2\pi)^{1/2}\alpha\hbar\omega_{LO}r_0\lambda_0. \tag{15}$$

We can obtain λ_0 by evaluating using the variational method. Then we can get the eigen level and the eigen wave-function. Then, we obtain the two-level system in a parabolic quantum dot the supposition electron is in system's ground state in the initial time, the electron will jump by the ground state to the excited state in presence of an electric field F along the x axis. Based on two level systems theory [11], the electron transition probability may express is:

$$p = \left(\frac{2\gamma}{\hbar\Omega}\right)^2 \sin^2\left(\frac{\Omega t}{2}\right), \tag{16}$$

where $\gamma = \langle\varphi_0|H'|\varphi_1\rangle = \langle\varphi_1|H'|\varphi_0\rangle$, $\Omega = \sqrt{\omega^2 + 4\gamma^2/\hbar^2}$, $\omega = (E_1 + \beta - E_0 - \alpha)/\hbar$, $\alpha = \langle\varphi_0|H'|\varphi_0\rangle$, $\beta = \langle\varphi_1|H'|\varphi_1\rangle$.

3 Results and Discussion

The numerical results of the electron transition probability versus the electric field, the electron-LO-phonon coupling constant and the confinement length in a parabolic QD are presented in the following figures. Throughout this study, the length and energy are taken in units of the polaron radius r_0 and the phonon energy constant $R^* = \hbar\omega_{LO}$.

Figure 1 shows the time evolution of the electron transition probability as a function of the electron-LO-phonon coupling constant for the strength of electric field $F = 10$, the confinement length $l_0 = 0.5$. It turns out that with increasing the electron-LO-phonon coupling constant, the electronic transition probability and the oscillation period gradually changes small. This is because the electron-LO-phonon coupling strength in the first-excited state is weaker than it in the ground-state with increasing the coupling strength. So the increasing of the energy spacing between the first-excited and ground state causes the reduction of the electron transition probability and the period of oscillation.

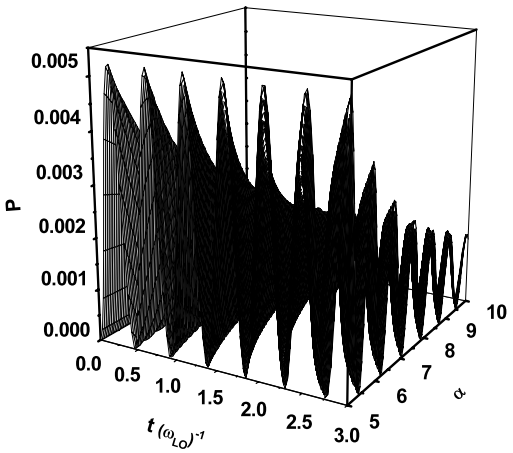

Fig. 1 The time evolution of the electron transition probability as a function of the electron-LO-phonon coupling constant

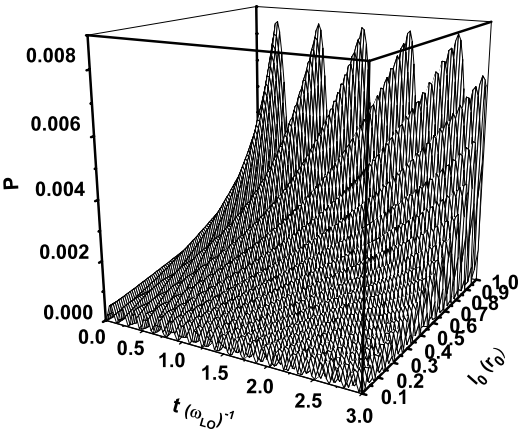

Fig. 2 The time evolution of the electron transition probability as a function of the confinement length of quantum dot

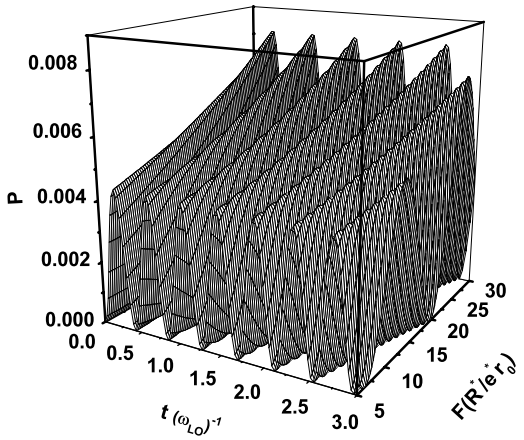

Fig. 3 The time evolution of the electron transition probability as a function of the electric field

Figure 2 presents that the time evolution of the electron transition probability as a function of the confinement length of quantum dot for the strength of electric field $F = 10$, the electron-LO-phonon coupling constant $\alpha = 6$. We can see that with increasing the confinement length, the electronic transition probability and the oscillation period increase quickly. This stem from the ground and the first-excited state energies are reduced with increasing the confinement length, but the influence is greater to the first-excited state energy than the ground state. For this reason, the energy spacing between the ground and the first-excited state decreases sharply, this also indicates the effect of quantum size.

Figure 3 plots the time evolution of the electron transition probability as a function of the strength of electric field for the confinement length of quantum dot $l_0 = 0.5$ and the electron-LO-phonon coupling constant $\alpha = 6$. From Fig. 3, one finds that the electronic transition probability and the oscillation period increase with increasing the electric field along the x direction. As a result of existing of the electric field, the ground and the first-excited state energies are reduced and the influence is greater to the first-excited state energy than the ground state. So the energy spacing between the ground and the first-excited state decreases.

4 Conclusion

The energies and the relevant eigen wave-functions of the ground and the first-excited state of electron have been obtained in a parabolic QD using the Pekar variational method considering the parabolic potential in the electron-LO-phonon strong coupling region. The single qubit can be envisaged as this kind of two-level quantum system in a QD. The supposition electron is in system's ground state in the initial time, the electron will jump by the ground state to the excited state in presence of an electric field F along the x axis. The results indicate that the electron transition probability and the oscillation period increase with decreasing the electron-LO-phonon coupling constant, increasing the electric field and the confinement length.

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